(**5 points**) Prove that ( Using Boolean expression)

**(45 points)** Simplify the function

* Using the Boolean expressions
* Using Karnaugh map
* Implement the simplified function using **NAND** and **Inverter** gates
* Implement the simplified function with a four-to-one multiplexer and external gates.
* Implement the simplified function with a 4-to-16 decoder and external gates.

**(15 points)** A digital circuit has a clock generator that produces pulses at a frequency of 125 MHz. Design a circuit that generates a clock with a frequency of 12.5 MHz.

(**35 points**) Show the complete gate level design of a state machine with two-bit input (**A1A0**). The state-machine should detect the sequence of **01**, **11**, and **01**,on its input. Provide a synchronous reset using external gates to start the machine in its initial state. The circuit produces a complete one-clock duration pulse on its **Y** output when the input sequence is detected. Show the circuit diagram using logic gates and **D flip-flops**. The machine should detect overlap of the sequence (If it receives the sequence of **01**, **11**, **01**, **11**, and **01**, the machine should detect 2 sequences).

(**Hint**: You must consider that when you have two inputs, on each arrow of the state machine you must consider the value of both inputs. In addition, for each state there are 4 different possibilities of transitions to other states)